

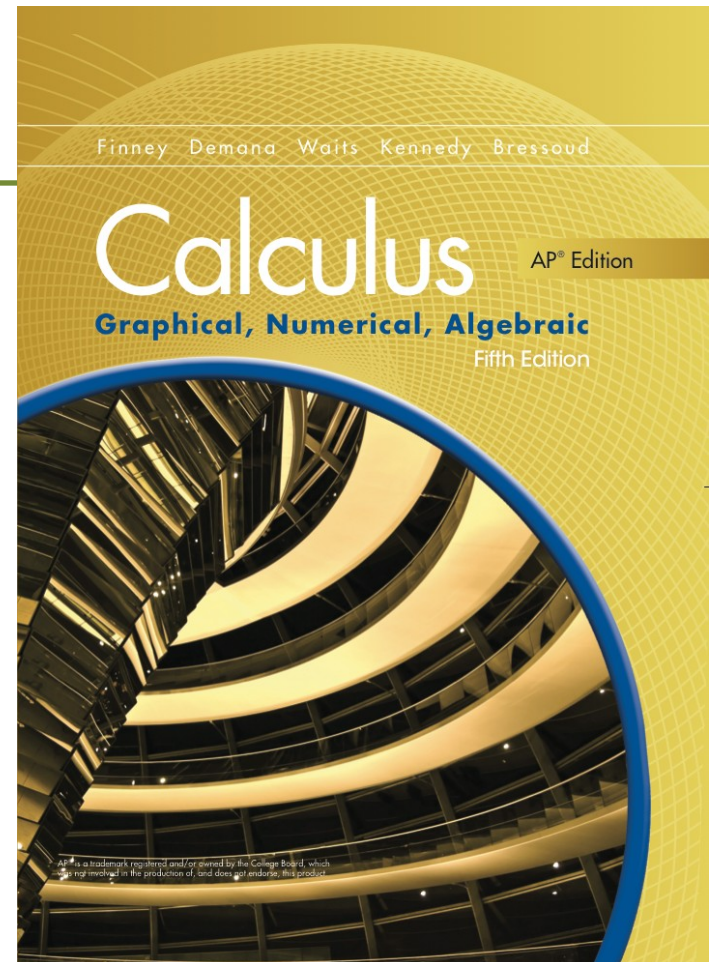
Chapter 6

Applications of Derivatives



Section 6.5

Trapezoidal Rule



Quick Review

Tell whether the curve is concave up or concave down on the given interval.

1. $y = \cos x$ on $[-1, 0]$

2. $y = x^4 - 3x^2 + 6$ on $[8, 17]$

3. $y = \sin\left(\frac{x}{2}\right)$ on $[48\pi, 50\pi]$

4. $y = e^{3x}$ on $[-5, 5]$

5. $y = 1/x$ on $[4, 8]$

6. $y = \csc x$ on $[0, \pi]$

7. $y = \sin x - \cos x$ on $[1, 2]$

Quick Review Solutions

Tell whether the curve is concave up or concave down on the given interval.

1. $y = \cos x$ on $[-1, 0]$ concave down

2. $y = x^4 - 3x^2 + 6$ on $[8, 17]$ concave up

3. $y = \sin\left(\frac{x}{2}\right)$ on $[48\pi, 50\pi]$ concave down

4. $y = e^{3x}$ on $[-5, 5]$ concave up

5. $y = 1/x$ on $[4, 8]$ concave up

6. $y = \csc x$ on $[0, \pi]$ concave up

7. $y = \sin x - \cos x$ on $[1, 2]$ concave down

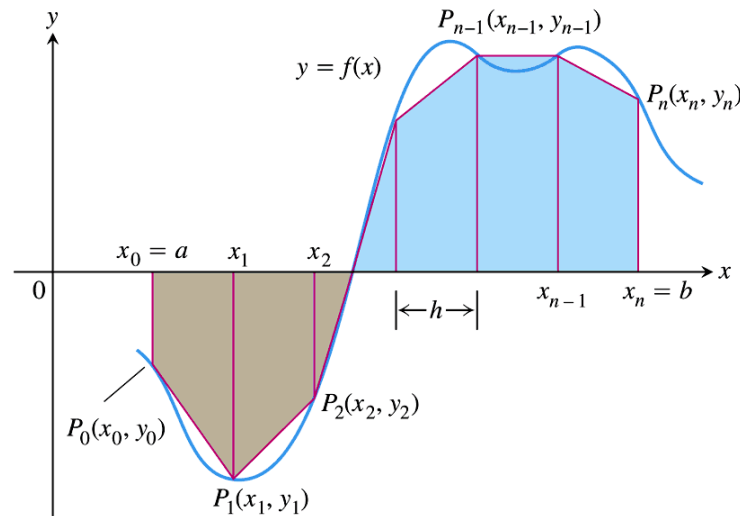
What you'll learn about

- Trapezoidal approximations
- Comparison to other numerical approximations
- Bounding the error in the Trapezoidal Rule

... and why

Some definite integrals are best found by numerical approximations, and rectangles are not always the most efficient figures to use.

Trapezoidal Approximations



$$\begin{aligned}\int_a^b f(x) dx &\approx h \cdot \frac{y_0 + y_1}{2} + h \cdot \frac{y_1 + y_2}{2} + \dots + h \cdot \frac{y_{n-1} + y_n}{2} \\ &= h \left(\frac{y_0}{2} + y_1 + y_2 + \dots + y_{n-1} + \frac{y_n}{2} \right) \\ &= \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n),\end{aligned}$$

where $y_0 = f(a)$, $y_1 = f(x_1)$, ..., $y_{n-1} = f(x_{n-1})$, $y_n = f(b)$.

The Trapezoidal Rule

To approximate $\int_a^b f(x) dx$, use

$$T = \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n),$$

where $[a, b]$ is partitioned into n subintervals of equal length

$$h = (b - a) / n.$$

Equivalently,

$$T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2},$$

where LRAM_n and RRAM_n are the Riemann sums using the left and right endpoints, respectively, for f for the partition.

Simpson's Rule

To approximate $\int_a^b f(x) dx$, use

$$S = \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{n-2} + 4y_{n-1} + y_n),$$

where $[a, b]$ is partitioned into an even number n subintervals of equal length $h = (b - a) / n$

Error Bounds

If T and S represent the approximations to $\int_a^b f(x) dx$ given by the Trapezoidal Rule and Simpson's Rule, respectively, then the errors E_T and E_S satisfy

$$|E_T| \leq \frac{b-a}{12} h^2 M_{ff} \quad \text{and} \quad |E_S| \leq \frac{b-a}{180} h^4 M_{(4)}$$

Quick Quiz Sections 6.4 and 6.5

You may use a graphing calculator to solve the following problems

1. The function f is continuous on the closed interval $[1, 7]$ and has values that are given below:

x	1	4	6	7
$f(x)$	10	30	40	20

Using the subintervals $[1, 4]$, $[4, 6]$, and $[6, 7]$, what is the trapezoidal approximation of $\int_1^7 f(x) dx$?

- (A) 110
- (B) 130
- (C) 160
- (D) 190
- (E) 210

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- (C) 160
- (D) 190
- (E) 210

Quick Quiz Sections 6.4 and 6.5

2. Let $F(x)$ be an antiderivative of $\sin^3 x$. If $F(1) = 0$, then $F(8) =$
- (A) 0.00
 - (B) 0.021
 - (C) 0.373
 - (D) 0.632
 - (E) 0.968

Quick Quiz Sections 6.4 and 6.5

2. Let $F(x)$ be an antiderivative of $\sin^3 x$. If $F(1) = 0$, then $F(8) =$

(A) 0.00

(B) 0.021

(C) 0.373

(D) 0.632

(E) 0.968

Quick Quiz Sections 6.4 and 6.5

3. Let $f(x) = \int_2^{x^2 - 3x} e^{t^2} dt$. At what value of x is $f(x)$ a minimum?

- (A) For no value of x
- (B) $1/2$
- (C) $3/2$
- (D) 2
- (E) 3

Quick Quiz Sections 6.4 and 6.5

3. Let $f(x) = \int_2^{x^2-3x} e^{t^2} dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x

(B) $1/2$

(C) $3/2$

(D) 2

(E) 3

Chapter Test

Let R be the region in the first quadrant enclosed by the x -axis and the graph of the function $y = 4x - x^3$.

1. Sketch the rectangles and compute by hand the area for the $MRAM_4$ approximations.
2. Sketch the trapezioids and compute by hand the area for the T_4 approximations.

Chapter Test

3. Suppose $\int_2^2 f(x) dx = 4$, $\int_2^5 f(x) dx = 3$, $\int_2^5 g(x) dx = 2$.

Which of the following statements are true, and which, if any, are false?

(a) $\int_5^2 f(x) dx = -3$

(b) $\int_2^5 [f(x) + g(x)] dx = 9$

(c) $f(x) \leq g(x)$ on the interval $-2 \leq x \leq 5$

4. Find the total area between the curve and the x-axis given $y = 4 - x$, $0 \leq x \leq 6$.

Evaluate using the Integral Evaluation Theorem.

5. $\int_0^1 (8s^3 - 12s^2 + 5) ds$

6. $\int_0^{\pi/2} \sec^2 \theta d\theta$

7. Evaluate: $\int_0^2 \frac{2}{y+1} dy$

Chapter Test

8. A diesel generator runs continuously, consuming oil at a gradually increasing rate until it must be temporarily shut down to have the filters replaced.

Day	Oil Consumption Rate (liters/hour)
Sun	0.019
Mon	0.020
Tue	0.021
Wed	0.023
Thu	0.025
Fri	0.028
Sat	0.031
Sun	0.035

- (a) Give an upper estimate and a lower estimate for the amount of oil consumed by the generator during that week.
- (b) Use the Trapezoidal Rule to estimate the amount of oil consumed by the generator during that week.

Chapter Test

9. Find dy/dx $y = \int_2^x \sqrt{2 + \cos^3 t} dt$

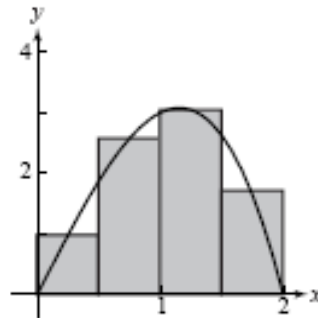
10. Solve for x : $\int_0^x (t^3 - 2t + 3) dt = 4$

Chapter Test Solutions

Let R be the region in the first quadrant enclosed by the x -axis and the graph of the function $y = 4x - x^3$.

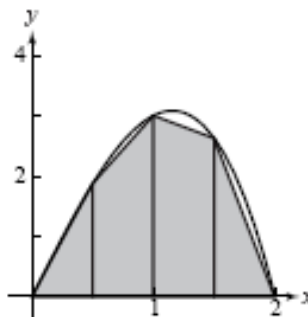
1. Sketch the rectangles and compute by hand the area for the MRAM₄ approximations.

4.125



2. Sketch the trapezioids and compute by hand the area for the T_4 approximations.

3.75



Chapter Test Solutions

3. Suppose $\int_2^2 f(x) dx = 4$, $\int_2^5 f(x) dx = 3$, $\int_2^5 g(x) dx = 2$.

Which of the following statements are true, and which, if any, are false?

(a) $\int_5^2 f(x) dx = -3$ True

(b) $\int_2^5 [f(x) + g(x)] dx = 9$ True

(c) $f(x) \leq g(x)$ on the interval $-2 \leq x \leq 5$ False

4. Find the total area between the curve and the x-axis given $y = 4 - x$, $0 \leq x \leq 6$. 10

Evaluate using the Integral Evaluation Theorem.

5. $\int_0^1 (8s^3 - 12s^2 + 5) ds$ 3

6. $\int_0^{\pi/2} \sec^2 \theta d\theta$ ∞

7. Evaluate: $\int_1^2 \frac{2}{y+1} dy$ $2\ln 3$

Chapter Test Solutions

8. A diesel generator runs continuously, consuming oil at a gradually increasing rate until it must be temporarily shut down to have the filters replaced.

Day	Oil Consumption Rate (liters/hour)
Sun	0.019
Mon	0.020
Tue	0.021
Wed	0.023
Thu	0.025
Fri	0.028
Sat	0.031
Sun	0.035

(a) Give an upper estimate and a lower estimate for the amount of oil consumed by the generator during that week. Upper = 4.392 L; Lower = 4.008 L

(b) Use the Trapezoidal Rule to estimate the amount of oil consumed by the generator during that week. 4.2L

Chapter Test Solutions

9. Find dy/dx $y = \int_2^x \sqrt{2 + \cos^3 t} dt$ $\sqrt{2 + \cos^3 x}$

10. Solve for x : $\int_0^x (t^3 - 2t + 3) dt = 4$ $x \approx 1.63052$ or $x \approx -3.09131$